# APPROXIMATE MODELING AND CALCULATION OF ARCLESS ELECTRIC SMELTING PROCESSES

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The rules of approximate modeling of the operating process of ore-thermic slag electric furnaces are worked out. A new method of engineering calculation of the main geometric and electric parameters of the indicated furnaces is proposed.

A general and simplified model of the operating process of an ore-thermic slag electric furnace in an arcless regime was proposed in [1, 2].*

The simplified model is reduced to a steady-state process of convective heat transfer governed by the presence of internal sources of Joule heat and ponderomotive forces in a viscous Newtonian fluid in an alternating sinusoidal electromagnetic field with appropriate boundary conditions.

The results of further simplification of the problem are given below, and the rules of approximate modeling and methods of calculating the main parameters of electric smelting furnaces are proposed.

In industrial furnaces the depth of penetration of the electro-magnetic wave into the slag melt is an order of magnitude greater than the linear dimensions of the region of spreading of the current, and therefore the skin effect and consequently the nonpotentiality of the electric field can be neglected. The integro-differential equations and modeling rules for this case were considered in [1, 2].

It is of interest to determine the effect of ponderomotive forces on the circulation of the slag melt. An analytical evaluation of the role of ponderomotive forces in the given case is difficult. Therefore, a special experiment was set up in which the ponderomotive effect was singled out from the total factors promoting circulation of the melt.

The idea behind the experiment was that in an electrolytic model of an electric slag furnace, after it is brought under a voltage, the ponderomotive effect should appear practically instantaneously, whereas the convection of heat should occur with some delay owing to the thermal inertia of the bath.

It is obvious that in the initial stage of this experiment the melt can be regarded as an isothermal medium in which heat release and heat transfer are absent. In this case the terms expressing the forces of gravity and static pressure, the sum of which is equal to zero, drop out of the equations of motion. The equation of motion in this case should reflect the equilibrium of the frictional (viscosity) forces, ponderomotive forces, electromagnetic pressure, and inertia.

The fundamental equations for this case in a steady regime are written so:

$$\operatorname{rot} \dot{\mathbf{e}} = 0; \ \operatorname{div} \dot{\mathbf{e}} = 0; \tag{44}$$

$$\rho (\mathbf{v}, \ \operatorname{grad}) \mathbf{v} = -\operatorname{grad} p + \mu \nabla^2 \mathbf{v} + \frac{1}{c_c^2} \sigma \operatorname{Im} [\dot{\mathbf{e}} \exp (j\omega\tau)] \times \int_F \frac{\sigma \operatorname{Im} [\dot{\mathbf{e}} \exp (j\omega\tau) \cdot \mathbf{l}^0}{l^2} dF; \tag{45}$$

*The present article is a continuation of [1, 2] and the enumeration of the equations is given in accordance with [1, 2].

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$$\operatorname{div} \mathbf{v} = 0. \tag{46}$$

The boundary conditions

$$\dot{u} = \frac{1}{\sqrt{2}} \int\limits_{l_m} \dot{\mathbf{e}} d\mathbf{l}_m; \tag{47}$$

on the boundaries with solid media

$$\mathbf{v} = 0; \tag{48}$$

on the free surface of the melt

$$p = 0. \tag{49}$$

An analysis of equations (44)-(49) leads to a functional relation of the determining and determinable dimensionless numbers in the general form:

$$\begin{gathered} \mathrm{Re} = \mathrm{Re}(\Pi) \equiv \mathrm{Re}(X,\ Y,\ Z,\ \mathbf{L}_{\mathrm{I}},\ \mathbf{L}_{\mathrm{II}},\ \ldots,\ \mathbf{L}_{i-1},\ \dot{U}); \\ K = K(\Pi);\ \mathrm{Eu} = \mathrm{Eu}(\Pi), \end{gathered} \tag{50}$$

where

$$X \equiv \frac{x}{l_i};\ Y \equiv \frac{y}{l_i};\ Z \equiv \frac{z}{l_i};\ \mathbf{L}_{\mathrm{I,II},\ldots,i-1} \equiv \frac{\mathbf{l}_{\mathrm{I,II},\ldots,i-1}}{l_i};$$

$$\dot{U} \equiv \frac{\dot{u} l_i \sigma \rho^{1/2}}{c_c \mu};\ \mathrm{Re} \equiv \frac{\mathbf{v} \mathbf{l}_i \rho}{\mu};\ K \equiv \frac{\sigma^2 \dot{\mathbf{e}}^2 \mathbf{l}_i^2}{c_c^2 \rho \mathbf{v}^2};\ \mathrm{Eu} \equiv \frac{p}{\rho \mathbf{v}^2}.$$

In accordance with the dimensionless relation (50) we fabricated a Plexiglas model of 1/10 natural size. As the model fluid we use a 25% aqueous solution of NaCl. The electrodes of the model were graphitized with a diameter of 100 mm. The experiment was conducted at $\dot{U} = 3.4 \cdot 10^3$, which exceeds the value of this complex for industrial furnaces.

After applying a voltage to the colored electrolytic cell, which was preliminarily held for a long time at room temperature, circulation of the electrolyte under the effect of ponderomotive forces was not observed at all for 1-2 sec. Then the electrolyte began to move intensely as a consequence of convection of heat.

Hence it follows that the pondermotive forces in a slag melt can be neglected.

Thus, after simplifications the investigated process reduces to a steady regime of free convection of the slag melt in the presence of internal distributed sources of Joule heat governed by the quasi-steady and potential electric field. The modeling roles for this case are presented in [3].

A further simplification of the problem is related with a determination of the role of molecular heat conduction in the process of heat transfer in the slag bath.

The physical parameters of the molten slag are characterized by the following quantities: $\lambda_* < 0.002$ kW/m·deg; $c_* > 0.8$ kW·sec/kg·deg; $\rho_* > 2000$ kg/m$^3$. The characteristic dimension of the bath $l_i > 0.5$ mm. For a velocity of the slag $v > 0.0001$ m/sec the Peclet number is

$$\mathrm{Pe} \equiv \frac{\lambda_*}{g c_* \rho_* v l_i} < 2.5 \cdot 10^{-2}.$$

Consequently the molecular heat conductivity must be taken into account at a slag velocity $v < 0.1$ mm /sec. According to the measurement data [3], such values of the slag velocity in the operating regime of the furnace are possible only in the laminar sublayer in the immediate vicinity of the boundaries with the solid media. We will estimate the thickness of the laminar sublayer and magnitude of the temperature gradient in it.

The useful heat flows from the slag melt to the solid charge in practice do not extend beyond the limits $q_{in} = 100$–$800$ kW/m$^2$. When $\lambda_* < 0.002$ kW/m·deg the normal temperature gradient in the melt at the interface with the solid charge should be accordingly

$$\frac{\partial t}{\partial n} = \frac{q_{in}}{\lambda_*} > 500\text{–}4000 \text{ deg/cm}.$$

In actuality, according to the results of direct measurements on furnaces and models [3] the temperature gradient in the slag melt at a distance of 1 cm from the interface with the solid charge is about 200 times less. Hence follows that the thickness of the laminar sublayer through which heat is transferred only by molecular heat conduction should be of the order 0.05 mm, and the temperature gradient on this thickness is accordingly 3-20°, which is less than the accuracy of measuring the temperatures in the melt (1600-1800°K).

It is obvious that under these conditions we can neglect the thickness of the laminar sublayer and temperature gradient in it, i.e., we can eliminate the laminar sublayer from the area of investigation.

In comparison with the useful heat flows, the flows of heat losses to the ambient medium through the lining and electrodes in industrial furnaces are insignificant and can be neglected also.

Thus for the indicated simplifications we can neglect molecular heat conductivity in the entire volume of the slag melt.

As is known, two regimes of free convection are possible: the viscosity—gravitation regime, the so-called "creeping" movement, in which we can neglect the inertial forces in comparison with viscosity forces, and the inertia—gravitation regime, in which we can neglect the viscosity forces in comparison with the inertial forces.

With consideration of the simplifications made, we will consider these two limiting cases on which is subsequently based the proposed method of calculating the main parameters of electric smelting furnaces.

I Case. A steady-state viscosity—gravitation regime of free convection in the presence of internal distributed Joule heat sources. We neglect the inertial forces and molecular heat conductivity. The physical parameters of the slag $\mu$, $\sigma$, $\rho$, and c depend on the temperature.

The fundamental equations:

$$\operatorname{rot}\dot{\mathbf{e}} = 0;\ \operatorname{div}(\sigma\dot{\mathbf{e}}) = 0; \tag{51}$$

$$\rho\mathbf{g} - \operatorname{grad} p + 2\operatorname{div}(\mu T_v) - 2/3\operatorname{grad}(\mu \operatorname{div}\mathbf{v}) = 0; \tag{52}$$

$$\operatorname{div}(\rho\mathbf{v}) = 0; \tag{53}$$

$$\frac{\sigma}{2}(\operatorname{Im}\dot{\mathbf{e}})^2 - \mathbf{v},\ \operatorname{grad}(c\rho g t) = 0. \tag{54}$$

The boundary conditions with respect to the electric field strength for the steady-state regime are substantiated in [1]:

$$\dot{u} = \frac{1}{\sqrt{2}}\int_{l_m}\dot{\mathbf{e}}\,d\mathbf{l}_m. \tag{55}$$

On the interface melt—wall, solid material and electrodes

$$v_n = 0; \tag{56}$$

on the free surface of the melt

$$j = 0. \tag{57}$$

The boundary conditions with respect to the temperature at the melt—charge interface

$$t = t_* = \text{const}; \tag{58}$$

on the interface melt—electrodes, walls

$$\frac{\partial t}{\partial n} = 0. \tag{59}$$

An analysis of Eqs. (51)-(59) leads to the functional dependence of the determining and determinable dimensionless numbers in the general form:

$$\Theta = \Theta(\Pi_1) \equiv \Theta(X,\ Y,\ Z,\ \mathbf{L}_{\mathrm{I}}\ \mathbf{L}_{\mathrm{II}},\ \ldots,\ \mathbf{L}_{i-1},\ \dot{U}_1);\ K_1 = K_1(\Pi_1);\ \mathrm{FrRe} = \mathrm{FrRe}(\Pi_1);\ \mathrm{Eu}:\mathrm{Fr} = \mathrm{Eu}:\mathrm{Fr}(\Pi_1), \tag{60}$$

where

$$X \equiv \frac{x}{l_i}; \quad Y \equiv \frac{y}{l_i}; \quad Z \equiv \frac{z}{l_i}; \quad \mathrm{L}_{\mathrm{I,II},\ldots,i-1} \equiv \frac{\mathrm{l}_{\mathrm{I,II},\ldots,i-1}}{l_i};$$

$$\dot{U}_1 \equiv \frac{\dot{u}}{\rho_* g l_i^{3/2}} \left( \frac{\mu_* \sigma_*}{c_* t_*} \right)^{1/2}; \quad \Theta \equiv \frac{t}{t_*}; \quad K_1 \equiv \frac{\dot{e}^2 \sigma \mathrm{l}}{\rho g c t \mathrm{v}};$$

$$\mathrm{FrRe} \equiv \frac{\rho g \mathrm{l}^2}{\mathrm{v} \mu}; \quad \mathrm{Eu}:\mathrm{Fr} \equiv \frac{p}{\rho g \mathrm{l}}.$$

The modeling rule consists in that in the model and specimen the determining dimensionless numbers $\mathrm{L_I}$, $\mathrm{L_{II}}$, . . . , $\mathrm{L}_{i-1}$, $\dot{\mathrm{U}}_1$ should be equal accordingly; the dimensionless dependences of the physical properties of the slag $\mu$, $\sigma$, $\rho$, and c on temperature (characteristic functions) should be accordingly identically the same.

As calculations and modeling experience shows, the indicated rule is easily fulfilled with an accuracy sufficient for practice in the case of a continuous covering of the melt by a solid charge and use of acidified glycerol (slag) and stearin (solid charge) as the model materials.

In this case the linear scale of the model can be selected sufficiently small, 1/10, and can be changed as the experimenter wishes within wide limits, since the condition $\dot{\mathrm{U}}_1$ = idem is easily fulfilled by an appropriate change of the electric strength on the electrodes of the model u.

II Case. A steady-state inertia–gravitation regime of free convection in the presence of internal distributed Joule heat sources. We neglect viscosity forces and molecular heat conductivity. The physical parameters of the slag $\sigma$, $\rho$, and c depend on the temperature.

Fundamental equations:

$$\mathrm{rot}\,\dot{\mathrm{e}} = 0; \quad \mathrm{div}\,(\sigma \dot{\mathrm{e}}) = 0; \tag{61}$$

$$\rho\,(\mathrm{v},\ \mathrm{grad})\,\mathrm{v} + \mathrm{grad}\,p - \rho \mathrm{g} = 0; \tag{62}$$

$$\mathrm{div}\,(\rho \mathrm{v}) = 0; \tag{63}$$

$$\frac{\sigma}{2}(\mathrm{Im}\,\dot{\mathrm{e}})^2 - \mathrm{v},\ \mathrm{grad}\,(c \rho g t) = 0. \tag{64}$$

The boundary conditions are the same as in variant I: Eqs. (55)-(59).

An analysis of (61)-(64) and (55)-(59) leads to the relation of the determining and determinable dimensionless numbers in the general form:

$$\Theta = \Theta(\Pi_2) \equiv \Theta(X,\ Y,\ Z,\ \mathrm{L_I},\ \mathrm{L_{II}},\ldots,\ \mathrm{L}_{i-1},\ \dot{U}_2),$$
$$K_1 = K_1(\Pi_2); \ \mathrm{Fr} = \mathrm{Fr}(\Pi_2); \ \mathrm{Eu} = \mathrm{Eu}(\Pi_2), \tag{65}$$

where

$$\dot{U}_2 \equiv \frac{\dot{u}}{(g l_i)^{3/4}} \left( \frac{\sigma_*}{\rho_* c_* t_*} \right)^{1/2}; \quad \mathrm{Fr} \equiv \frac{g\mathrm{l}}{\mathrm{v}^2}; \quad \mathrm{Eu} \equiv \frac{p}{\rho \mathrm{v}^2}.$$

The other dimensionless numbers are the same as in variant I. The modeling rule consists in that in the model and specimen the determining dimensionless numbers $\mathrm{L_I}$, $\mathrm{L_{II}}$, . . . , $\mathrm{L}_{i-1}$, $\dot{\mathrm{U}}_2$ should be equal accordingly; the dimensionless dependences of the physical properties of the slag on temperature (characteristic function) should be accordingly identically the same.

The indicated rule is fulfilled with an accuracy sufficient for practice in the case of a continuous covering of the melt by a solid charge and use of salted water (slag) and stearin (charge) as the model materials. In this case the linear scale of the model, as in variant I can be selected sufficiently small, 1/10, and can be varied as the investigator wishes by changing the voltages on the electrodes $\dot{\mathrm{u}}$.

The complexes $\dot{\mathrm{U}}_1$ and $\dot{\mathrm{U}}_2$ represent dimensionless voltages on the electrodes of the furnace (model). To use these complexes in engineering calculations it is expedient to find the expressions for the dimensionless resistance and power of the furnace.

In a potential and quasi-steady electric field the resistance of a medium of any configuration as the ratio of the voltage to the current can be expressed in the following way:

$$r_e = \int_{l_m} e_{l_m} dl_m ; \int_s e_s \sigma ds. \tag{66}$$

An analysis of (66) leads to a functional relation of the determinable (dimensionless electrical resistance $R_e$) and determining dimensionless numbers in the general form:

$$R_e = R_e(L_I, L_{II}, \ldots, L_{i-1}, \Sigma), \tag{67}$$

where

$$R_e \equiv r_e \sigma_* l_i; \; \sum \equiv \frac{\sigma}{\sigma_*}; \; L_{I, II, \ldots, i-1} \equiv \frac{l_{I, II, \ldots, i-1}}{l_i}.$$

On fulfilling the modeling rules formulated above for the I or II case the conditions $\Sigma$ = idem and $L_{I, II, \ldots, i-1}$ = idem and consequently $R_e$ = idem will also be fulfilled. This permits transforming the modeling conditions with respect to the dimensionless voltages $\dot{U}_1$ = idem and $\dot{U}_2$ = idem to the modeling conditions with respect to the dimensionless powers respectively for the I case

$$W_I \equiv \frac{\dot{U}_1^2}{R_e} = \frac{w\mu_*}{(\rho_* g)^2 c_* t_* l_i^4} = \text{idem}; \tag{68}$$

for the II case

$$W_2 \equiv \frac{\dot{U}_2^2}{R_e} = \frac{w}{\rho_* c_* t_* g^{3/2} l_i^{5/2}} = \text{idem}. \tag{69}$$

In this case the relation between the voltage u and the power w is determined from the equations $\dot{U}_{1(2)}$ = idem and is expressed by the relations: for the I case

$$u = a_1 \left( \frac{c_* t_*}{\mu_*} \right)^{0.125} \frac{(\rho_* g)^{0.25}}{\sigma_*^{0.5}} w^{0.375}, \tag{70}$$

for the II case

$$u = a_2 \frac{(\rho_* c_* t_*)^{0.2} g^{0.3}}{\sigma_*^{0.5}} w^{0.3}. \tag{71}$$

The shape factors $a_1$, $a_2$, which are constant for geometrically similar slag baths, are determined from the experience of operating industrial furnaces by means of (70) or (71).

Thus for the assumptions made in accordance with the aforementioned modeling rules with observance of geometric similarity of the slag baths and identical equality of the dimensionless dependences of the physical properties on temperature (characteristic functions) and accordingly of conditions (68) or (69), the electrical, temperature, velocity, and pressure fields in the slag baths of comparable furnaces will be the same.

For the same slag melting points in comparable furnaces ($t_*$ = idem) the condition of similarity of temperature fields, expressed by the equation

$$\Theta \equiv \frac{t}{t_*} = \text{idem},$$

degenerates to the condition of equality of temperatures (t = idem) at similar points of the slag baths and, in particular, at the boundaries with the furnace lining.

The conditions of identical equality of the dimensionless characteristic functions and melting points of the slags ($t_*$ = idem) are fulfilled exactly on comparing furnaces of the same technology in which the same charges are processed and slags that are the same in composition and physical properties are obtained.

In this case conditions (68), (70) and (69), (71) are simplified considerably and are transformed to the form: for the I case

$$l_i = b_1 w^{0.25}, \tag{72}$$

$$u = b_1' w^{0.375}, \tag{73}$$

for the II case

$$l_i = b_2 w^{0.4}, \tag{74}$$

$$u = b_2' w^{0.3} . \quad (75)$$

The coefficients $b_1$, $b_1'$, $b_2$, $b_2'$ are constant for a given composition of the charge and slag and are determined from the experience of operating industrial furnaces by means of the relations (72), (73) or (74), (75), respectively.

From the practical data the operating process of industrial electric slag furnaces is described satisfactorily by relations (68), (70), (72), and (73) for the I case for a value of the dimensionless power $W_1 > 5 \cdot 10^{-9}$ and by relations (69), (71), (74), and (75) for the II case with $W_1 < 5 \cdot 10^{-9}$. Here the cube root of the size of the volume of the subelectrode slag layer corresponding to one electrode is taken as the determining dimension $l_i$.

We will give an example confirming the correspondence of the results obtained to the practical data.

An empirical formula was obtained in [14] which generalizes the practical data on industrial phosphorus electric furnaces within the charge $w_f$ = 12-50 MW:

$$\psi = 285 + 11.4 w_f - 0.04 w_f^2. \quad (76)$$

Calculation shows that for these furnaces $W_1 \approx 10^{-8}$ and consequently for comparison we must use (72) for the I case. From (72)

$$\psi = b_3 w_f^{0.5}. \quad (77)$$

For a value of the constant $b_3 = 110$ in (77) the calculation results by Eqs. (76) and (77) practically coincide.

By means of relations (74) and (75) for the II case we calculated the geometric parameters and voltage during intensification of the operation of the industrial electric furnace for processing tin-containing raw material, for which $W_1 \approx 10^{-9}$. With an increase of furnace output by 1.5 times the temperature regime and degree of wear of the furnace scarcely changed.

The results obtained permit proposing the new method of calculating the main geometric and electrical parameters of the ore-thermic electric slag furnace being designed.

The existing calculation methods are based on the condition of the equality of the specific surface electric power ($\psi$ = idem) in the "standard" furnace and furnace being designed. However, it is known from practice that during operation it is possible to increase considerably the technical and economic indices of the electric slag furnace without noticeably shortening the service life of the furnace lining by increasing the specific power of the furnace in comparison with the design power.

The proposed method of calculation, unlike the existing ones, is based on the more concrete and physically substantiated condition of equality of temperatures at identical points at the interfaces of the melt with the lining in the "standard" furnaces and furnaces being designed (t = idem) and is consistent with the practical data on intensification of electric smelting.

A furnace employing the new technology can be calculated by Eqs. (68) or (69) and (70) or (71) in the same way, but with a greater error (as a consequence of the inequality of the dimensionless characteristic functions and melting points $t_*$). It is expedient to compensate the inaccuracy of the calculation by selecting additional voltage steps of the furnace transformer above and below the design value.

## NOTATION

$\dot{\mathring{a}}$ is the maximum value of the complex vector of quantity $a$;
$\dot{a}$ is the complex value of quantity $a$;
$\mathbf{a}$ is the vector $a$;
$c_c$ is the speed of light in a vacuum;
$j$ is the imaginary unit;
$\omega$ is the angular frequency of alternating current;
$\tau$ is the time;
$\sigma$ is the specific electric conductivity;
$e$ is the electric field strength;
$u$ is the effective voltage between electrode and zero point in melt (phase voltage);
$t$ is the temperature;

$t*$ is the melting point of slag (at liquidus point);
$g$ is the acceleration of gravity;
$v$ is the velocity of melt;
$p$ is the pressure;
$\lambda$ is the molecular heat conductivity;
$c$ is the specific heat per unit weight;
$\rho$ is the density;
$\mu$ is the coefficient of dynamic viscosity;
$x, y, z$ are the linear coordinates;
$l_I, l_{II}, \ldots, l_i$ are the geometric parameters;
$l_m$ is the distance between the electrode and zero point in the melt in any direction;
$l$ is the radius vector whose start is at the current point and end is at a given point of space;
$l^0$ is the unit radius vector;
$F$ is the volume of melt;
$\partial t/\partial n$ is the temperature gradient along the normal to the interface;
$q_{in}$ is the quantity of heat passing through a unit surface of the interface in unit time;
$T_v$ is the strain rate tensor;
$s$ is the equivalent surface of the electric field in the melt;
$r_e$ is the electric resistance of the melt reduced to one electrode;
$w$ is the electric power generated in melt per electrode;
$w_f$ is the total power generated in melt of furnace, MW;
$\psi$ is the specific surface power of furnace, i.e., power generated in melt per unit area of furnace hearth, $kW/m^2$.

Subscripts

n normal components of the vector;
* physical parameters of the slag at $t*$.

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